Sensitivity Analysis of the IMAGE Greenhouse Model

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Abstract Sensitivity analysis is an important component of model building as it provides information about the parameters which have major influences on the model and its outputs. IMAGE (Integrated Model to Assess the Greenhouse Effect) has gained acceptance as a mid-range model for working with and predicting climate change. It is an ideal model for the application of new methods of sensitivity analysis. The sensitivity of IMAGE is analysed to include first-order and second-order effects. This includes a first estimate of the non-linear interactions between the parameters of IMAGE. The results of these analyses are presented and some of the important first- and second-order interactions are identified. The strengths of these interactions indicate where the modelling of processes incorporated in IMAGE needs to properly represent reality.

1. INTRODUCTION

The natural phenomenon of the *Greenhouse Effect* is responsible for the Earth's surface temperature suitable for life. The nature of this phenomenon can be explained by the analysis of energy and carbon fluxes in the Earth-atmosphere system. In order for the climate system to remain in a steady state, the balance between incoming and outgoing energy (radiation) must be maintained, otherwise the Earth would continue to get warmer or colder. This balance can be disturbed by changes in the magnitude of the Greenhouse Effect caused by anthropogenic emission of gases and aerosols. Recent studies have focused on the Greenhouse Effect and the possible climate changes which may be induced by changing concentrations of the Greenhouse gases in the atmosphere. Several mathematical and numerical models have been developed trying to assess these effects. IMAGE 1.0 (Integrated Model to Assess the Greenhouse Effect) is a PC based Greenhouse Model incorporating carbon and heat cycles on a global scale. It has been developed at RIVM [Rotmans, 1990], and is a precursor of IMAGE 2.0 [Alcamo *et al.*, 1994].

Sensitivity analysis has been carried out on IMAGE 1.0 by Wiseman *et al.*, [1995]. A number of parameters were selected among the large number of parameters contained in the model (approximately 250) and Sensitivity Analysis experiments were performed using Factorial Experimentation [Daniel 1976, Box *et al.*, 1978]. Results of the experimental runs were then analysed using a Table of Contrasts [Daniel, 1976, Henderson Sellers, B., 1992] and by Analysis of Variance [Wiseman *et al.*, 1995].

Here, Sensitivity Analysis has been performed again on IMAGE 1.0 following the development of the Morris method [Morris, 1991], and its extension by Campolongo and Gabric [1997], and Campolongo and Braddock [1997]. The modified Morris method is designed to minimise the number of model evaluations required to obtain sensitivity information. Three state variables and six input parameters were selected for this study. Results have been compared with the ones previously obtained by Wiseman *et al.* [1995].

2. THE MODEL

IMAGE 1.0 is a one-dimensional integrated model of climate change based on globally averaged variables. It has a modular structure combining relatively simple models of global carbon cycle (atmosphere, biosphere, and the ocean), atmospheric chemistry, radiative forcing, and heat exchange in the ocean. They are linked with the energy use, emission and land use modules and temperature change, sea level rise and socio-economic impact modules. The driving force of the model is human-induced greenhouse gas emissions, resulting either from direct emissions or emissions caused by land use changes (deforestation). Those emissions plus the carbon flux between the ocean and the atmosphere determine the total greenhouse gas radiative forcing of the atmosphere. Radiative forcing and the energy flux from the deep oceans determine the actual temperature of the surface mixed ocean layer. The global temperature change is caused by the temperature rise of the mixed ocean layer.

From a mathematical point of view, IMAGE 1.0 is a sequence of first order differential equations, and ordinary algebraic equations, which are solved using Runge-Kutta numerical algorithm. In particular, IMAGE 1.0 is formulated as a control dynamic system of the form [Braddock *et al.*, 1994]

$$\frac{d}{dt}\mathbf{X}(t) = \mathbf{F}(\mathbf{X}(t) + \mathbf{U}(t)$$
$$\mathbf{X}(t_0) = \mathbf{X}_0 \qquad (2.1)$$

where the time $t \in (1990,2100)$, t_0=1990 is the initial simulation time (symbolising the end of the pre-industrial area), $\mathbf{X}(t) \in \mathbf{R}^{155}$ is a vector of state variables, $\mathbf{U}(t) \in \mathbf{R}^{155}$ is the forcing term or human interference term, and $\mathbf{F}(\mathbf{X}(t)): \mathbf{R}^{155} \to \mathbf{R}^{155}$ describes the climate system processes. The solution $\mathbf{X}(t)$, as a function of time, is called a trajectory of (2.1). The forcing term $\mathbf{U}(t)$ includes, implicitly, scenarios of future population growth, fossil fuel combustion, deforestation, and technology development in the period 1990-2100.

The forcing term in IMAGE 1.0 corresponds to one of the four standard IPCC 1990 scenario [Houghton *et al.*, 1990] devised by Rotmans and colleagues at RIVM. The four IPCC scenarios are: *Business as Usual (BaU)*, in which the present trend of unrestricted use of natural resources continues in the next century; *Forced Trends (FT)*, an environmentally oriented scenario; *Reduced Trends (RT)* and *Changed Trends (CT)* scenarios, falling between these two extremes.

From a computational point of view, IMAGE 1.0 is a deterministic computer simulation model. Each model evaluation takes 3-5 minutes on a 486 PC. To facilitate its use, the model has been expanded into WIM (Windows for IMAGE), a software system designed to provide a framework for combining global climate models such as IMAGE 1.0 with local or regional models of the impacts of climate change. WIM is based on a friendly graphical user interface (GUI). GUI allows the user to set up simulations, with different scenarios and parameters values, and analyse the results making use of all the printing and graphics facilities of Microsoft Windows 3.1.

3. SENSITIVITY ANALYSIS

Of the 155 state variables of IMAGE 1.0, three were selected in this sensitivity analysis exercise. Those variables are:

- Total sea level rise (meters),
- Atmospheric CO_2 concentration (parts per million by volume, ppmv),
- Temperature change of the ocean mixed layer ($°C$).

Those state variables belong to three different modules of IMAGE. The three modules are receptively: the Sea Level Rise module, the Atmospheric Carbon Cycle module, and the Ocean Circulation module. In this study, a limited number of six input parameters were selected, and these are:

1. Biotic growth factor, which is a dynamic parameter;
2. Initial Atmospheric Concentration of CO_2, an initial condition in year t=1990;
3. Rate of Increase of the Net Primary Production (NPP), a dynamic parameter,
4. Initial Atmospheric Concentration of Methane, an initial condition in year t=1990;
5. Initial Atmospheric Concentration of Nitrous Oxide, an initial condition in year t=1990;
6. Equilibrium Temperature Increase due to Doubling of CO_2, a parameter representing a flow between modules.

In order to be able to compare results with others previously obtained, the ranges of variation for the six parameters were assumed to be equal to the ones selected by Wiseman *et al.* [1995]. Those ranges of variations, which are ±10% of the "base" parameter value (i.e. the value "from reference"), are given in Table 1.

Table 1: Ranges of variation selected for the six model parameters listed above.

Factor	Base value	Minimum value	Maximum value
1	0.3	0.272	0.332
2	296	266.1	320.0
3	1.4	1.26	1.54
4	0.9	0.81	0.99
5	280	252.1	308.1
6	2.5	2.26	2.75

4. THE EXPERIMENTAL PLAN

The sensitivity analysis was performed on IMAGE 1.0 using the extended Morris method [Campolongo and Braddock, 1997]. The new version of the method, still varying one-factor-at-a-time, (as in the original version proposed by Morris [1991]), allows the investigator to estimate not only the "main" effects, but also the higher order effects due to interaction among pairs of parameters. The computational cost (C) of the extended Morris method of the sensitivity experiment is

$$C = r \times \{\{(k+1) + [(k-1) \times (k/2-1)]\} + (k-1) \times k/2\},$$

where k is the number of model parameters selected for the analysis, and r is the size selected for the samples of Elementary Effects. Note that $C < r \times k^2$.

In this exercise, the number of parameters is k=6, and the selected sample size is r=4. Thus, the cost of the sensitivity experiment is $C=4\times\{7+10+5\times3\}=128$.

The six parameters selected for testing the model sensitivity, were sampled in the parameter space with a sampling step $\Delta=1/3$ (of the real interval in which the parameter is varying), i.e. this is a 4-level experiment.

5. RESULTS

Tables 2 and 3 show results of the sensitivity analysis obtained for the four possible scenarios. In those Tables, for each of the three output variables under study, the values of the Morris sensitivity measures (μ and σ), and the measures of the two-term interactions effects, are given for each of the six selected input parameters. In Table 4, for each scenario and for each output variable, the most significant effects are shown in descending order.

5.1 The Total Sea Level Rise

The sensitivity analysis shows the Total Sea Level Rise to be mainly sensitive to the equilibrium temperature increase due to carbon dioxide doubling, and to the initial atmospheric concentration of nitrous oxide, for any possible scenario, although the order of significance is not necessarily the same. Furthermore, the Business as Usual scenario, shows the initial concentration of atmospheric CO_2 to be the most important parameter. Those results obtained for the Total Sea Level Rise agree with those previously obtained by Wiseman *et al.* [1995], using the fractional factorial design.

In Wiseman *et al.* [1995], where a fractional factorial design was used instead of a full design, two-term interaction effects tend to be not clearly identified. In fact, those effects are either confounded among each other or with three factor interactions. In our experiment, an order of importance for the two-factor interaction effects is provided. Of those effects, the most significant on the Total Sea Level Rise, are the ones relative to the pairs of factors 6-4 (equilibrium temperature increase due to carbon dioxide doubling - initial atmospheric concentration of methane), 1-3 (biotic growth factor - rate of increase of NNP), and 6-2 (equilibrium temperature increase due to carbon dioxide doubling - initial atmospheric concentration of CO_2) (see Table 3).

5.2 The Concentration of Carbon Dioxide

In agreement with what has already been discovered by using the Fractional Factorial design, the concentration of carbon dioxide has shown to be

Table 2: One-Factor Sensitivity Means and Standard Deviations

Factor	Business as usual		Reduced trends		Changed trends		Forced trends	
	μ	σ^2	μ	σ^2	μ	σ^2	μ	σ^2
				TOTAL SEA-LEVEL RISE				
1	7.32E-03	2.05E-05	6.95E-03	1.93E-05	7.07E-03	1.92E-05	6.07E-03	1.46E-05
2	**2.72E-02**	**3.08E-04**	1.59E-02	1.02E-04	1.18E-02	5.52E-05	5.90E-03	1.28E-05
3	1.22E-02	6.31E-05	1.20E-02	6.23E-05	1.19E-02	6.08E-05	1.13E-02	5.44E-05
4	1.05E-02	3.97E-05	1.18E-02	4.96E-05	1.25E-02	5.66E-05	1.32E-02	6.22E-05
5	**1.49E-02**	**8.03E-05**	1.61E-02	9.39E-05	**1.65E-02**	**1.00E-04**	**1.70E-02**	**1.08E-04**
6	**7.95E-02**	**2.46E-03**	**5.68E-02**	**1.25E-03**	**5.07E-02**	**1.00E-03**	**3.74E-02**	**5.44E-04**
				ATMOSPHERIC CO_2 CONCENTRATION				
1	**1.88E+01**	**1.29E+01**	**1.06E+01**	**4.19E+01**	**8.65**	**2.80E+01**	**5.68**	**1.21E+01**
2	**1.44E+02**	**8.05E+03**	**1.23E+02**	**5.89E+03**	**1.17E+02**	**5.33E+03**	**1.07E+02**	**4.39E+03**
3	**3.33E+01**	**4.58E+02**	**1.97E+01**	**1.60E+02**	**1.56E+01**	**1.01E+02**	**1.15E+01**	**5.58E+01**
4	6.99E-01	4.63E-01	7.00E-01	2.84E-01	2.25E-01	1.27E+01	7.50E-01	3.35E-01
5	9.75E-01	4.77E-01	8.75E-01	2.53E-01	1.87	1.23E+01	9.00E-01	2.62E-01
6	5.35	1.13E+01	3.27	3.39	2.82	2.87	2.05	1.31
				TEMPERATURE CHANGE IN MIXED LAYER DEPTH				
1	**6.04E-01**	**2.63**	2.00E-01	7.82E-01	7.40E-02	**1.98E-01**	4.93E-03	**1.04E-01**
2	2.38E-01	2.45E-02	1.23E-01	6.37E-03	7.92E-02	2.47E-03	3.63E-02	4.98E-04
3	**3.63E-01**	**2.72**	**3.49E-01**	**7.46E-01**	**2.71E-01**	**1.82E-01**	**1.73E-01**	**9.48E-02**
4	1.98E-01	1.75	8.63E-02	2.74E-03	9.05E-02	2.98E-03	9.15E-02	2.98E-03
5	1.23E-02	1.29	1.21E-01	5.45E-03	**1.22E-01**	**5.65E-03**	**1.22E-01**	**5.68E-03**
6	**7.73E-01**	**3.78**	**4.45E-01**	**7.95E-02**	**3.73E-01**	**5.51E-02**	**2.63E-01**	**2.73E-02**

Table 3: Two-Factor Interaction Sensitivities

Factors	Business as usual λ_{ij}	Reduced trends λ_{ij}	Changed trends λ_{ij}	Forced trends λ_{ij}
TOTAL SEA-LEVEL RISE				
1 2	4.32E-02	2.60E-02	2.14E-02	1.15E-02
1 3	**7.06E-02**	4.48E-02	3.65E-02	2.12E-02
1 4	3.62E-02	2.25E-02	1.87E-02	1.03E-02
1 5	**5.27E-02**	4.18E-02	**3.89E-02**	**3.33E-02**
1 6	**5.98E-02**	4.84E-02	**4.59E-02**	**3.87E-02**
2 3	2.50E-02	1.10E-02	1.39E-02	1.75E-02
2 4	8.55E-03	9.23E-03	1.01E-02	9.22E-03
2 5	4.70E-02	3.24E-02	2.86E-02	2.07E-02
2 6	**7.40E-02**	**0.12**	3.58E-02	1.87E-02
3 4	1.01E-02	9.00E-03	7.88E-03	7.20E-03
3 5	1.87E-02	1.82E-02	1.91E-02	1.80E-02
3 6	2.74E-02	2.63E-02	2.77E-02	2.61E-02
4 5	1.33E-02	1.08E-02	1.28E-02	6.75E-03
4 6	**8.75E-02**	6.21E-02	**5.47E-02**	**3.92E-02**
5 6	**5.15E-02**	3.60E-02	3.24E-02	2.36E-02
ATMOSPHERIC CO_2 CONCENTRATION				
1 2	8.10	5.40	5.63	3.82
1 3	**1.22E+02**	1.10E+02	1.08E+02	9.92E+01
1 4	9.22	5.18	4.50	2.70
1 5	1.17E+01	6.75	5.85	4.28
1 6	8.55	4.95	3.83	2.47
2 3	4.59E+01	2.70E+01	2.14E+01	1.55E+01
2 4	**1.33E+02**	**1.17E+02**	**1.13E+02**	**1.04E+02**
2 5	1.10E+01	6.75	5.40	3.60
2 6	**1.35E+02**	**1.18E+02**	**1.13E+02**	**1.04E+02**
3 4	**1.34E+02**	**1.18E+02**	**1.14E+02**	**1.05E+02**
3 5	9.63E+01	9.54E+01	9.49E+01	9.07E+01
3 6	**1.28E+02**	**1.13E+02**	1.09E+02	**1.01E+02**
4 5	9.69E+01	9.58E+01	1.04E+02	9.09E+01
4 6	**1.06E+02**	1.01E+02	1.09E+02	9.41E+01
5 6	1.21E+01	6.75	5.85	3.83
TEMPERATURE CHANGE IN MIXED LAYER DEPTH				
1 2	**4.61**	**2.89**	**1.30**	**8.98E-01**
1 3	**4.86**	**2.63**	**1.40**	**9.65E-01**
1 4	3.28E-01	1.82E-01	1.06E-01	7.43E-02
1 5	4.70E-01	3.31E-01	2.95E-01	2.36E-01
1 6	1.17	3.80E-01	3.38E-01	2.77E-01
2 3	1.44E-01	7.87E-02	1.06E-01	1.22E-01
2 4	8.55E-02	7.20E-02	8.32E-02	6.75E-02
2 5	4.21E-01	2.59E-01	2.09E-01	1.46E-01
2 6	1.31	3.62E-01	2.56E-01	1.28E-01
3 4	7.65E-02	6.08E-02	4.95E-02	4.50E-02
3 5	1.60E-01	1.35E-01	1.42E-01	1.35E-01
3 6	**4.47**	**2.47**	**1.34**	**1.01**
4 5	1.05	7.88E-02	5.85E-02	4.50E-02
4 6	1.77	4.79E-01	3.91E-01	2.70E-01
5 6	4.64E-01	2.90E-01	2.43E-01	2.79E-01

mainly affected by the atmospheric concentration of carbon dioxide in 1990 and the rate of increase of NPP.

Of the two-factor interaction effects, the following four are recognised as important: (1) interactions between atmospheric concentration of carbon dioxide in 1990 and the equilibrium temperature increase due to

Table 4

Variable / Scenario	Total Sea Level Rise	Temperature Change in Ocean Mixed Layer	Concentration of Carbon Dioxide
Business as usual	2(B) 6(G) 5(E) 4-6(DG) 1-3(AC) 2-6(BG)	6(G) 1(A) 3(C) 1-2(AB) 1-3(AC) 3-6(CG)	2(B) 3(C) 1(A) 2-6(BG) 3-4(CD) 2-4(BD) 3-6(CG)
Reduced trends	6(G) 5(E) 2-6(BG)	6(G) 3(C) 1-3(AC) 1-2(AB) 3-6(CG)	2(B) 3(C) 1(A) 2-6(BG) 3-4(CD) 2-4(BD) 3-6(CG)
Changed trends	5(E) 6(G) 1-5(AE) 1-6(AG) 4-6(DG)	6(G) 3(C) 5(E) 1-2(AB) 1-3(AC) 3-6(CG)	2(B) 3(C) 1(A) 3-4(CD) 2-6(BG) 2-4(BD)
Forced trends	6(G) 5(E) 4-6(DG) 1-6(AG) 1-5(A-E)	6(G) 3(C) 5(E) 3-6(CG) 1-3(AC) 1-2(AB)	2(B) 3(C) 1(A) 3-4(CD) 2-6(BG) 2-4(BD) 3-6(CG)

carbon dioxide doubling, (2) interactions between atmospheric concentration of methane in 1900 and the rate of increase of the NPP, (3) interactions between atmospheric concentration of methane in 1900 and the equilibrium temperature increase due to carbon dioxide doubling, (4) interactions between the rate of increase of the NPP and the equilibrium temperature increase due to carbon dioxide doubling. The order in the significance of those effects is slightly different from one scenario to another.

5.3 The Temperature Change of the Ocean Mixed Layer

The results also indicate that temperature change of the ocean mixed layer is mainly sensitive to the equilibrium temperature increase due to carbon dioxide doubling, thus confirming results obtained by Wiseman *et al.* [1995]. This parameter turned out to be the most important one for each of the four scenarios. The rate of increase of the NPP is also strongly affecting the temperature change. This parameter is the second important one for three of the scenarios (i.e. RT, CT and FT scenarios), and the third most important one for the Business as Usual scenario. Wiseman *et al.* [1995] identify this parameter as significant, only in the Forced Trend scenario.

The initial atmospheric concentration of nitrous oxide is identified by Wiseman *et al.* [1995] for the Changed and Forced Trends. Also, results of this experiment suggest this parameter to be important in those scenarios and not in the others.

The only two-term interaction effect recognised as significant by Wiseman *et al.* [1995] for each of the four scenarios is that involving the rate of increase of NPP and the equilibrium temperature increase due to carbon dioxide doubling. In particular, in the FT scenario, this is the only two-factor effect identified. The SA method employed here confirms this result. In the FT scenario, this effect is identified as the main one; in the others, it is regarded as an important effect, (although not the main one), together with the effects due to the interactions of the biotic growth factor, initial CO_2 concentration and rate of increase of NPP.

6. DISCUSSION

The sensitivity analysis for the IMAGE model has been carried out only relating to a small subset of the full set of parameters and initial values used in WIM. Analysing the sensitivity of the three selected output variables, to variations in all of the model input factors, would result in a sensitivity experiment with an enormous computational cost.

Only a relatively small range of variation (± 10% of the mean value) was chosen for each of the selected input factor. This was because the range was chosen to be the same of the one used in Wiseman *et al.* [1995], so that results could be compared.

Sensitivity analysis experiments on IMAGE have also been performed by Rotmans [1990]. However, in that experiment, instead of taking into consideration the whole model, single modules were tested in turn. This was possible because the modules forming IMAGE are relatively self contained, and communicate to each other through a limited set of state variables and some associated parameters. Nevertheless, it is more appropriate to compare these results with the ones obtained by Wiseman *et al.* [1995] by running the whole model.

Results shown in Section 5 are in good agreement with the ones previously obtained by using the Fractional Factorial design [Wiseman *et al.*, 1995], confirming that future studies should aim to improve measurements of the identified parameters. The method employed here, has the advantage, with respect to the Fractional Factorial method, of providing more information. In fact, this method also provides an order of importance for the effects due to interactions between two factors, the so-called two-factor interaction effects. Those effects are confounded among each other or with third order effects in the analysis carried out by Wiseman *et al.* [1995].

As a drawback, our method has a higher computational cost (128 runs were performed against the 32 needed by the Fractional Factorial design). However, it should be pointed out that this is due to the low number of input parameters selected. In fact, while the computational cost of the extended Morris method is $O(k^2)$, the computational cost of performing a factorial experiment increases exponentially (2^k for a full factorial design). Utilisation of a Fractional Factorial design then becomes essential, and when, in order to reduce the computational cost, the degree of fractionation is increased, the resolution of the design becomes lower, i.e. confounding the effects of various order increases.

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